*Journal of the Society of Automotive Engineers Malaysia*
*Volume 2, Issue 3, pp 289-305, September 2018*
*e-ISSN 2550-2239 & ISSN 2600-8092*

# Linear Quadratic Regulator Based Control Device for Active Suspension System with Enhanced Vehicle Ride Comfort

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**ORIGINAL ARTICLE** ***Open Access***

**Article History:**

Received
*27 Feb 2018*

Received in revised form
*10 Aug 2018*

Accepted
*15 Aug 2018*

Available online
*1 Sep 2018*

**Abstract** – *The suspension system is required in an automobile in order to absorb shock that comes from various type of disturbances such as irregular road profile, engine vibrations and wheel. Besides, the suspension plays an important part in enhancing the passenger ride comfort. The suspension will make sure that the tire is always contacted with the road for a better grip and braking. Conventionally, passive suspension has been used in car manufacturing that leads to huge vibrations that affect the ride quality. This is because ride comfort of passengers gets affected by overshoot and settling time of vehicle under vibration. Therefore, a good controller design is required to minimize the vibrations. In this research, an active suspension of quarter car model that considers only vertical movement is utilized in the suspension system. This paper presents a Linear Quadratic Regulator (LQR) method to enhance the vehicle ride comfort towards the vibration of the suspension system. The control design approach is then compared with the classical control which is the Proportional Integral Derivative (PID) that is set as a benchmark control. The results for both controllers are evaluated through simulations in MATLAB and Simulink Software. Other than using the passenger vehicle parameters, the parameters of bus are also tested into the system to investigate the vehicle performance by taking the bumps and road pavements as road disturbances. The results obtained from the simulations show that the responses of the quadratic based approach give the significant improvement in minimizing the vibration and fast settling time compared to passive and PID control.*

**Keywords:** Active suspension system, quarter car model, PID control, LQR control, ride comfort

## 1.0 INTRODUCTION

The automotive industry uses different kinds of wheel or axle suspension systems. The important criteria that need to be concerned are costs, space requirements, kinematic properties and compliance attributes. The targets of a suspension system of a vehicle are to carry the car and its weight, keep the tires in contact with the road as well as control the vehicle's direction of travelling. The shock absorbing is needed in an automobile suspension system. On the other hand, the main causes of vehicle body and suspension system vibration are pot holes and speed bumps (Yakub et al., 2017). After a road impact is applied, a vibrational oscillation will be subjected to the wheels and transmitted to their axle. Therefore, a better suspension system is needed for better ride quality as the comfort of passengers gets affected through overshoot and settling time.

Springs and dampers are the basic suspension components that come along with other parts and they are used to form various types of suspension units utilized in different cars. Different suspension systems are set up for back and front wheels depending on the type of vehicles being used. The most crucial part in vehicle that influences especially the ride comfort of the passengers is the suspension system, where at the same time, another important part is to ensure the ride safety. (Matsumoto et al., 1999). A detailed experimental work on passenger comfort has been conducted by Griffin (1996) to resolve the effects of road excitation. Shuaishuai et al. (2013) investigated the mechanism of the train vibration. Various degree of freedoms of critical speeds with respect to damping rates and the stiffness of suspension were analysed so that the safety and passenger comfort can be achieved.

Generally, there are three types of suspension system commonly known as passive, semi-active and active suspension systems. Traditional suspension system in car manufacturing is the passive suspension system in which the fixed sets of spring and damper are located parallel between the mass of car body and the wheel mass. Through their constant characteristics, there is only open loop system involved in which traditional suspension does not require any feedback control mechanism. (Rosli et al., 2014; Abtahi, 2017). In other words, the passive suspension system cannot control the damping and stiffness and can only make certain operating conditions to get an optimum damping effect. Thus, it is difficult to adapt from the various road conditions to achieve the satisfactory ride comfort as well as to handle stability problem.

In order to gain a good suspension system, the semi-active suspension system was then introduced and studied by the researchers. Huang and Xu (2015) made the improvements in fuzzy logic control in semi-active vibration control system. They implemented the particle swarm optimization to limit current in the case of the sensitive model used to predict the output through experimental and simulations process. In order to produce accurate mechanical model of MR-damper, Hu et al. (2017) conducted experiments and proposed new hybrid fuzzy and fuzzy proportional integral derivative approaches to enhance the ride comfort. The advantages of using the semi active suspension system in car are it requires low cost and a lot less energy. In active suspension system, there is an actuator placed in parallel to the shock absorbers and the spring that allows the enhancement of the passenger comfort (Alvarez-Sánchez, 2013). Then, the development of semi-active suspension system was introduced, which combines the benefit of passive and active suspension systems respectively (Prabakar et al., 2009).

Numerous researches have been performed throughout the world on active suspension system. The nonlinear behavior of hydraulic system and uncertain parameters in active suspension system has increased the difficulty in creating mathematical model for active suspension system. Therefore, an adaptive neuro fuzzy inference system was designed by Senthil et al. (2018) to handle actuator dynamics and parameter uncertainty in hydraulic actuator. Zhang and Gong (2018) designed a robust passive fault tolerant control scheme based on $H_2/H_\infty$ and integral sliding mode passive fault tolerant control scheme based on $H_2/H_\infty$ to improve the ride comfort of the active suspension system. In another study, Riaz and Khan (2017) presented a nonlinear full car active suspension system with seated driver biodynamics. To get the low frequency of vibrations, they utilized a neuro-fuzzy adaptive paradigm in the vehicle model. Their findings were compared to passive and conventional control strategy. Taskin et al. (2017) used an active suspension of quarter car model in the analysis of reducing the displacement and acceleration magnitudes of sprung mass. Vaughan et al. (2003) had discussed the use of active suspension control on how to solve the effects of vehicle payloads.

The improvement of vehicle suspension control has gained a lot of attentions from automobile industrial researchers and academics. Many control approaches have been proposed and designed to meet the requirements of active suspension systems in order to obtain good ride comfort and road holding performances. These include PID control (Graa et al., 2017), Fuzzy Logic (FL) control (Khan & Qamar, 2013), Sliding Mode Control (SMC) (Yao et al., 2013) and skyhook, ground hook and hybrid control (Ahmadian et al., 2000).

Throughout this paper, two controller designs of active suspension systems will be proposed to improve the ride comfort of passengers which are PID and LQR control system. The PID based approach has been widely used in many control applications including cars and lorry due to its simple mechanism and has the ability of gain tuning. Swati and Jain (2013) studied the application of Proportional Integral (PI) and PID controller to give out smooth bus suspension system by reducing the overshoot and vibrations. The settling time for the oscillations to reach its stability was also measured for open-loop, PI and PID respectively. The simulations were carried out through Matlab and Simulink software and the results were then compared for responses on open loop, PI and PID. Bello et al. (2015) performed a study in enhancing the ride comfort as well as driving stability by designing a double loop PID on half car model through Matlab and Simulink software. Yakub et al. (2016) conducted a study on enhancing the ride comfort by setting a PID controller as a benchmarking control for a comparison with fuzzy logic approach.

Furthermore, most of the studies are carried out to investigate the ability of LQR control performance in suspension system in which it can be concluded that LQR strategy can provide better ride performance. Rao and Kumar (2014) designed a LQR control in semi-active suspension system and change the hydraulic damper to magnetorheological damper for controlling the damping force. A study conducted by Amer et al. (2014) utilized LQR control in electromagnetic suspension system. The controller performance revealed that LQR strategy gave an improvement in passenger comfort compared to conventional suspension system. The primary used of LQR is to minimize the cost function. Hence, due to the advantages of LQR control, the control design of PID and LQR will be implemented in this paper to enhance the passenger comfort at given scenario of disturbance inputs.

The main objective of this study is to improve the ride comfort by designing a suitable controller which is the LQR approach that is capable of reducing the vibration and getting the optimum settling time. The behavior of the controller performance is analysed based on the quarter car active vehicle suspension model through the simulation process in Matlab and Simulink software. As the contribution of this paper, the response of the controller performance is investigated by taking speed bumps with different height and width, also the road pavements as the input variables. The vehicle parameters of bus and sedan car are considered in this study because these two types of vehicles give different vehicle dynamic behavior.

This paper is organized as followed. The modelling of quarter car active suspension is illustrated in Section 2. Section 3 gives the controller designs of the benchmark controller which uses PID control and the proposed controller which applies LQR control that is expected to reach the criteria needed in reducing the vibration in active suspension system. The results of the designed controller as compared to the passive and LQR are discussed in Section 4. The future work and conclusion are explained in Section 5.

## 2.0 MODELLING OF QUARTER CAR ACTIVE SUSPENSION SYSTEM

The active suspension system for quarter car model is represented in the form of mathematical model. The modelling of active suspension system is described in differential equation and state space model. By using the Newton's Law equation of motions, the active suspension for quarter car model can be developed. In this section, the two degrees of freedom of quarter car model is designed as shown in the Figure 1. The vertical motion of sprung mass is represented by $m_1$, while $m_2$ is denoted for unsprung mass. The disturbance signal corresponds to $w$, which represents a bump and $x_1$ represents car displacement of the car body meanwhile $x_2$ gives the displacement of the wheel. The spring constants are denoted as $k_1$ and $k_2$ whereas $b_1$and $b_2$ represent dampers. Finally, $u$ is the control force.

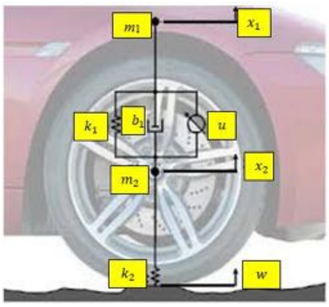



**Figure 1:** Quarter car model

All the equations of motion can be obtained by drawing a free body diagram based on the mathematical model in the Figure 1. Figure 2(a) and 2(b) illustrate the free body diagram of the sprung and unsprung masses, respectively.

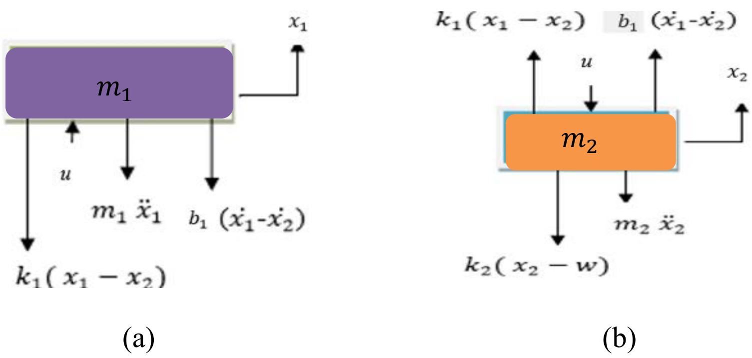



**Figure 2:** (a) Free body diagram of car body; (b) Free body diagram of wheel mass

From the free body diagrams above, the equations can be formed for both $m_1$ and $m_2$ as shown in the equation (1) – (6).

$$m_1 \ddot{x}_1 = - k_1( x_1 - x_2) - b_1 (\dot{x}_1 - \dot{x}_2) + u \tag{1}$$

$$m_2 \ddot{x}_2 = - k_1( x_1 - x_2) - b_1 (\dot{x}_1 - \dot{x}_2) + k_2( x_2 - w) - u \tag{2}$$

The equations of motion of the quarter car model for active suspension are given by the following state-space representation. The state-space equation is created by using the following state variables.

$$X_1 = x_1 - x_2,\ X_2 = \dot{x}_1 ,\ X_3 = x_2 - w ,\ X_4 = \dot{x}_2 \tag{3}$$

The general equations from (3) are written as followed by taking the equations from (1) and (2). The state space representation is as in the following form where t is the continuous time variable. The passenger vehicle parameters are shown in the Table 1 and Table 2 for car and bus respectively.

$$\dot{X}(\mathrm{t}) = \mathrm{A}X(\mathrm{t}) + \mathrm{B}u(\mathrm{t}) + f(\mathrm{t}) \tag{4}$$

$$\dot{X}_1 = \dot{x}_1 - \dot{x}_2 \text{ (suspension deflection)} \qquad \dot{X}_2 = \ddot{x}_1 \text{ (car body acceleration)}$$

$$\dot{X}_3 = \dot{x}_2 - \dot{w} \text{ (wheel deflection)} \qquad \dot{X}_4 = \ddot{x}_2 \text{ (wheel acceleration )} \tag{5}$$

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \\ \dot{X}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & -1 \\ \frac{-k_1}{m_1} & \frac{-b_1}{m_1} & 0 & \frac{b_1}{m_1} \\ 0 & 0 & 0 & 1 \\ \frac{-k_1}{m_2} & \frac{b_1}{m_2} & \frac{-k_2}{m_2} & \frac{-b_1}{m_2} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m_1} \\ 0 \\ \frac{1}{m_2} \end{bmatrix} u + \begin{bmatrix} 0 \\ 0 \\ -1 \\ 0 \end{bmatrix} \dot{w} \tag{6}$$

**Table 1:** Passenger vehicle parameter (Agharkakli et al., 2012)

| Symbol | Description | Value |
|---|---|---|
| $m_1$[kg] | Sprung mass | 290 |
| $m_2$[kg] | Unsprung mass | 59 |
| $k_1$[n/m] | Spring constant of suspension system | 16,812 |
| $k_2$[n/m] | Spring constant of wheel and tire | 190,000 |
| $b_1$[n/m/s] | Damping constant of suspension system | 1,000 |

**Table 2**: Bus parameter (Yakub et al., 2017)

| Symbol | Description | Value |
|---|---|---|
| $m_1$[kg] | Sprung mass | 2,500 |
| $m_2$[kg] | Unsprung mass | 320 |
| $k_1$[n/m] | Spring constant of suspension system | 80,000 |
| $k_2$[n/m] | Spring constant of wheel and tire | 500,000 |
| $b_1$[n/m/s] | Damping constant of suspension system | 350 |

## 3.0 CONTROLLER DESIGN

### 3.1 PID Controller

There are many researches that have been conducted using PID controller especially for quarter car model active suspension system. The PID controller is the most used feedback control design while showing the three terms operating on the error signal to produce a control signal. For the quarter car active suspension system, a controller is designed to reduce the vibration of the suspension and the deflection of vehicle. The objective of the controllers is to reduce the vibration of car suspension system while increasing the ride comfort of passenger and driver. PID controller used PID tuner to get values of gain which are proportional gain, integral gain and derivative gain. PID control is chosen as the practical control because the user can choose from many options of large numbers in which it helps the user in changing the dynamics of the system and also it is easy to be tuned (Giovanni, 2009). In order to minimize the vibration in a suspension system, the PID equation is used as stated below:

$$u(t) = K_p\, e(t) + K_i \int_0^t e\,(\tau) d\tau + K_d \,{}^{de(t)}\!/_{dt} \qquad (7)$$

From equation (7), $K_p$ is denoted as the proportional coefficients, $K_i$ is integral coefficients, $K_d$ is derivative coefficient and the error signal is represented by $e$. In this work, several tunings based on the settling time and percentage of overshoot have been measured for simulation purposes in Matlab and Simulink software. The method of finding the values of proportional, derivative and integral are through PID tuner until the best response is obtained (Figure 3).

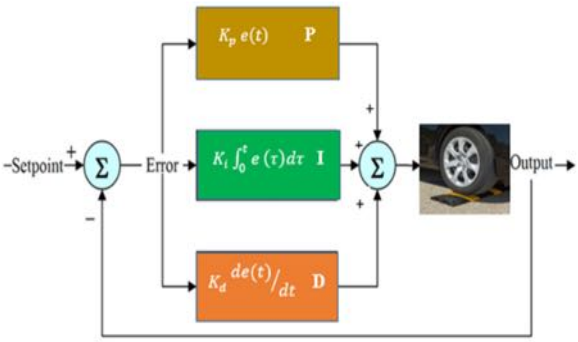



**Figure 3:** PID controller design

### 3.2 LQR Controller

The Linear Quadratic Regulator (LQR) is another type of controller which regulates the system and drives the initial state vector to zero. The implementation of LQR in this linear system is to improve the ride comfort for a quarter car model. The state variable regulator for the system is considered as:

$$u(t) = K\, x(t) \qquad (8)$$

where $K$ is the constant state feedback gain matrix $(m \times n)$ that gives the closed loop state equation with desired performance characteristics. Basically, the main function of this controller is to be able to minimize the cost function, $J$ and finally calculate the optimal gain, $K$.

$$J = \frac{1}{2}\int_0^t (x^T Qx + u^T Ru) \quad (9)$$

*J* is the parameter index that must be minimized by balancing the *Q* and *R* matrices. From (9), *u* is the input which is the multiplication of gain *K* whereas *Q* and *R* are the optimality in the optimal control law.

$$K = R^{-1}B^T P \quad (10)$$

The *P* matrix must fulfil the reduced-matrix Ricatti equation given as:

$$A^T P + PA - PBR^{-1}B^T P + Q = 0 \quad (11)$$

Then, the feedback regulator *u* is:

$$u(t) = -(R^{-1}B^T P)\, x(t) \quad (12)$$

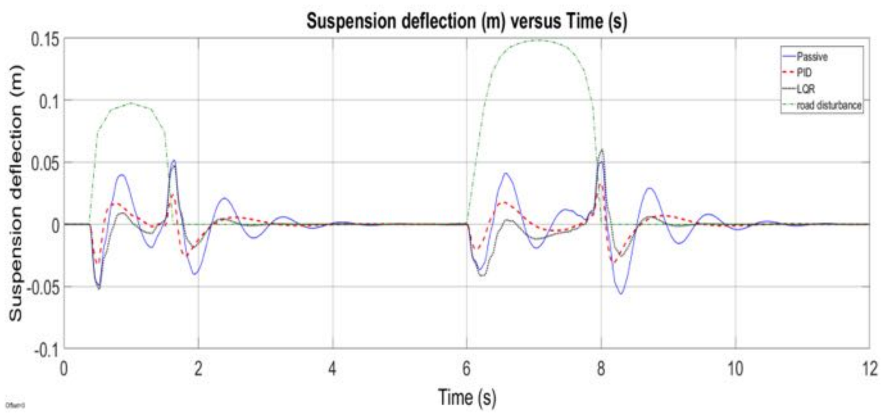
$$u(t) = -K\, x(t) \quad (13)$$

## 4.0 RESULTS AND DISCUSSION

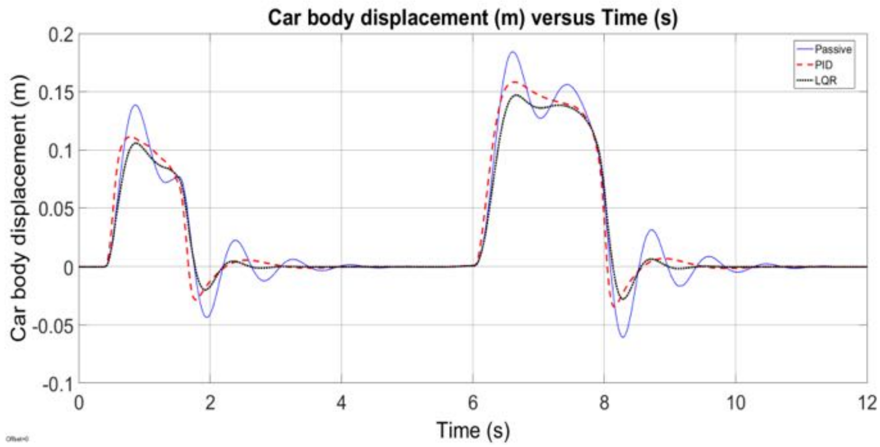
### 4.1 Scenario Description

This section is divided into two parts. First, the performances of the LQR, PID and passive controllers are dealt with car parameters followed by bus parameters through simulations in Matlab and Simulink software. The performances of the controllers are observed at the present of bumps and after bumps. This action is also implemented when the road pavement is taken as the other input variable. The parameters that are monitored for both scenarios in this paper are suspension deflection, car body displacement and wheel deflection for a car and a bus. The quarter car model is assumed to be at a constant speed along the road within 12 seconds and 20 seconds for car and bus respectively. There will be a road disturbance utilized in the system which is dual bumps with different heights which are 10 cm and 15 cm. The first bump will start at 0.5 second whereas the second bump starts at time 8 seconds for both tested vehicles.

### 4.2 Simulations for Bump Cases




(a)

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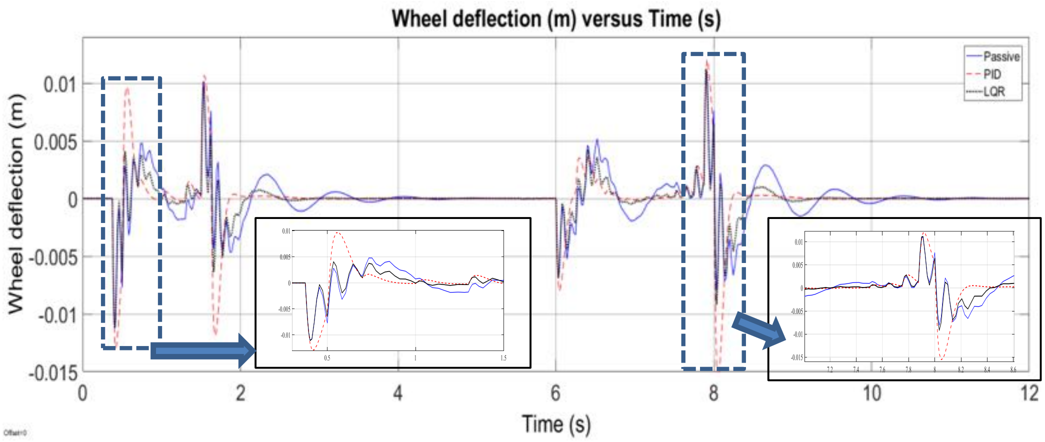
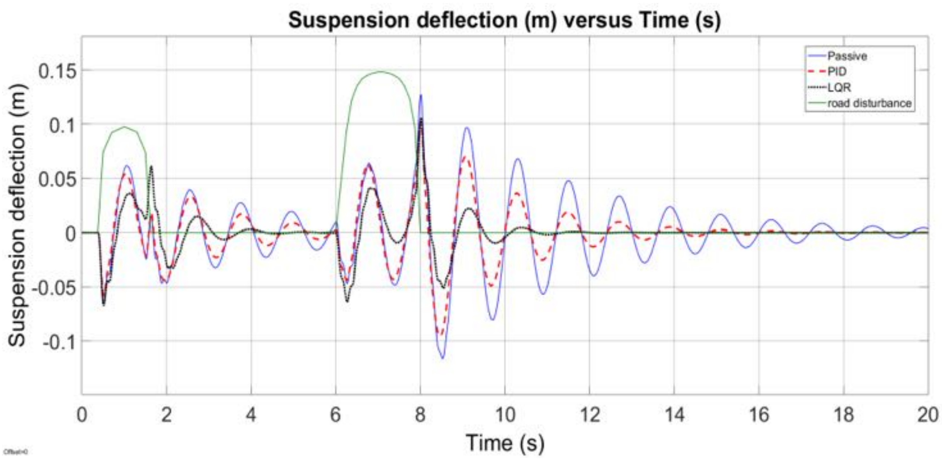



(c)

**Figure 4**: Vehicle performances of a car: (a) Suspension deflection; (b) Car body displacement; (c) Wheel deflection with magnified view

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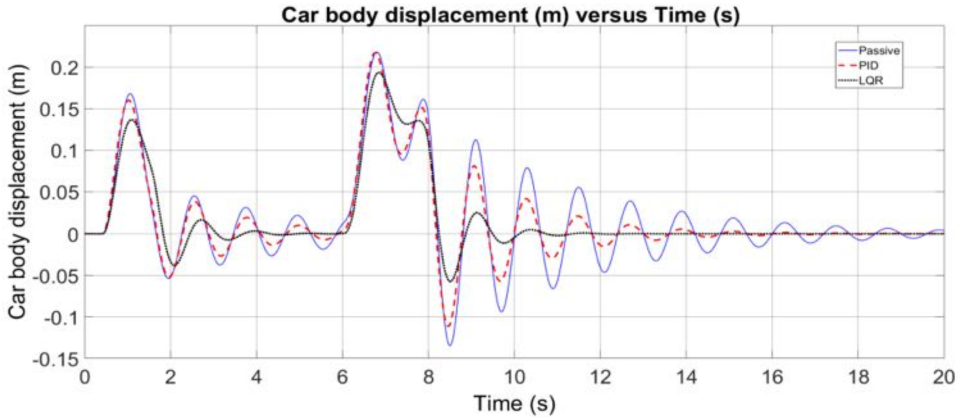


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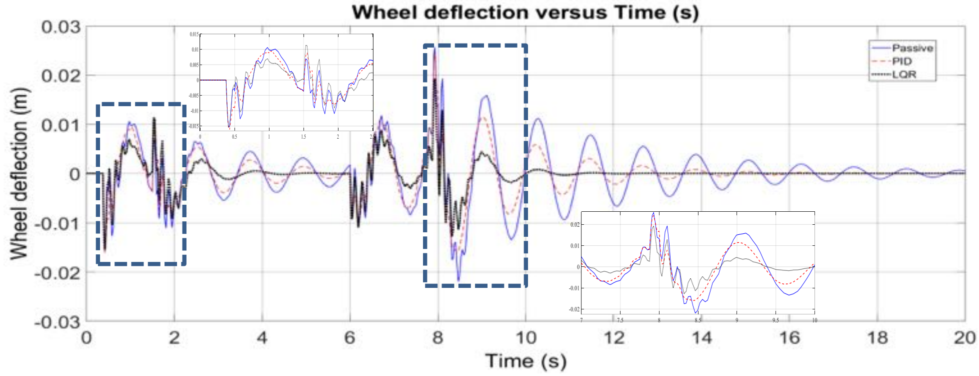



(c)

**Figure 5**: Vehicle performances of a bus: (a) Suspension deflection; (b) Car body displacement; (c) Wheel deflection with magnified view

The responses of the quarter car suspension system were obtained through Matlab and Simulink software. Figure 4 shows the comparison between suspension deflection, car body displacement and wheel deflection of car over passive, PID and LQR controls for dual bumps with different heights. Meanwhile, the output response for bus is indicated in Figure 5. In utilizing the LQR approach for car and bus, the matrix $Q$ and $R$ are used through trial and error approach such as:

Car:

**LQR**

$$Q = \begin{bmatrix} 79460 & 0 & 0 & 0 \\ 0 & 1379 & 0 & 0 \\ 0 & 0 & 52118 & 0 \\ 0 & 0 & 0 & 5000 \end{bmatrix} \text{ and } R = 0.01 \qquad K_p = 25 \;\; K_i = 1000 \;\; K_d = 35$$

and gain $K$ = [2058 1155 1240 1496].

Bus:

**LQR** **PID**

$$Q = \begin{bmatrix} 6500 & 0 & 0 & 0 \\ 0 & 50000 & 0 & 0 \\ 0 & 0 & 20000 & 0 \\ 0 & 0 & 0 & 2500 \end{bmatrix} \text{ and } R = 0.01 \qquad K_p = 55 \;\; K_i = 950 \;\; K_d = 65$$

and gain $K$ = [ 40 5830 -1.891 724]

Figure 4(a) and Figure 5(a) demonstrate the spring behaviour when hitting the dual bumps for the given time at the constant speed. The results show that the passive suspension gives the higher vibration and takes the longest time to reach the stable state. The dual bump condition affects the suspension deflection for both car and bus. For suspension deflection with car parameters, the amplitude for LQR control is slightly higher compared to PID control. However, LQR control gives the best output response in achieving faster settling time. Meanwhile with bus parameters, the amplitude and the settling time using the LQR approach is able to reduce the vibration greatly compared to passive and PID control. Since the bumps are located near to each other, the vibration is completely reduced after hitting the second bump. As presented in Figure 4(a) for car, the oscillations manage to stop before the present of the second bump. Meanwhile in Figure 5(a) for bus, the vibrations on passive and PID are still oscillating after the present of the second bump and they manage to reach their steady state even though it takes time. The oscillations are reduced greatly after utilizing the LQR control. This can be seen from the graph that, LQR control manages to optimize the vibration already at the first bump before meeting the second bump.

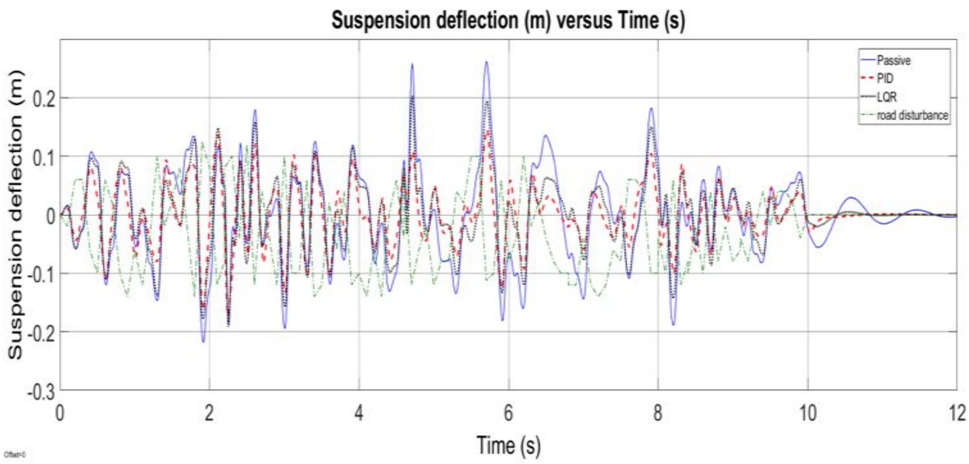
Figure 4(b) and 5(b) show the comparison between body displacements of car and bus. For output responses to car parameter, the PID and LQR methods were able to reach the steady state, almost at the same time. By referring to the graph of bus parameter, the conventional passive suspension system has the highest vibration and still oscillates when hitting the second bump. LQR controller that is represented by the black dotted line is capable of minimizing the amplitude of vibration greatly, compared to PID controller. Based on the simulation results from the respected figures, the vibrations on passive and PID are still oscillating after the present of the second bump and they managed to reach their steady state even though it takes time. This phenomenon illustrates that the passenger or driver will still feel uncomfortable due to the occurrence of vibrations in the vehicle suspension system. Even though the bus and the car gave different vehicle dynamic behaviour, as shown in the respected figures, the graph of LQR control gave significant improvement that is capable of reducing the vehicle body displacement over passive and PID suspension system. These results prove that the LQR control provides a good ride comfort compared to the PID suspension control.

The simulation results for wheel deflection of bus and car are presented in Figure 4(c) and 5(c). The results indicate that LQR control has the best performance as it presents the reduction of the vibration greatly of the wheel deflection. The amplitude of wheel deflection in LQR control is lower than that of the PID control for bus and car parameters. By implementing the PID control into the system, the amplitude for output response in car parameter at the first bump is slightly higher compared to the reference input and the LQR control. However, the amplitude as well as the settling time are optimized by using the LQR approach. Meanwhile for bus parameters, the amplitude of LQR control that is presented by black dotted line has minimized the vibration as compared to those of the PID and the conventional suspension control system.

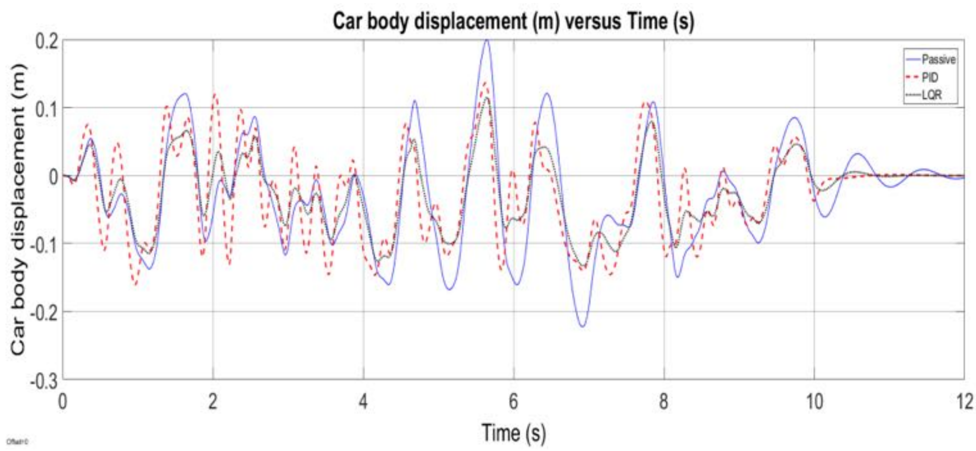
Looking in-depth into the sensitivity of the parameters, the scenario is changed when the road pavement is considered as the input variable. Figure 6(a) and Figure 7(a) demonstrate the spring behaviour when the system hits the road pavements at constant speed. The results show that the passive suspension leads to higher vibration compared to PID and LQR control for bus and car parameters, respectively. The significant improvement of vibration reduction can be seen through LQR control. In Figure 6(b) and 7(b) show the comparison in body displacement corresponds to car and bus, respectively. From the results, it can be seen clearly that LQR control provides the greatest reduction of amplitude when compared to passive and PID approaches.

As shown in the respected Figure 6(c) and 7(c), the graph of LQR control gives the significant improvement that is able to reduce the vibration of wheel deflection over passive and PID suspension systems. These results prove that LQR control provides better performance compared to PID suspension control.

### 4.3 Simulations for Road Pavements




(a)

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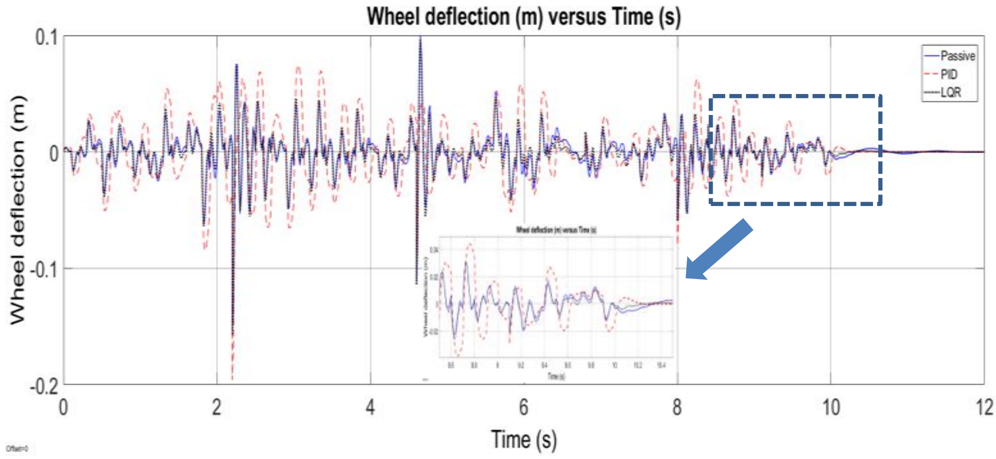
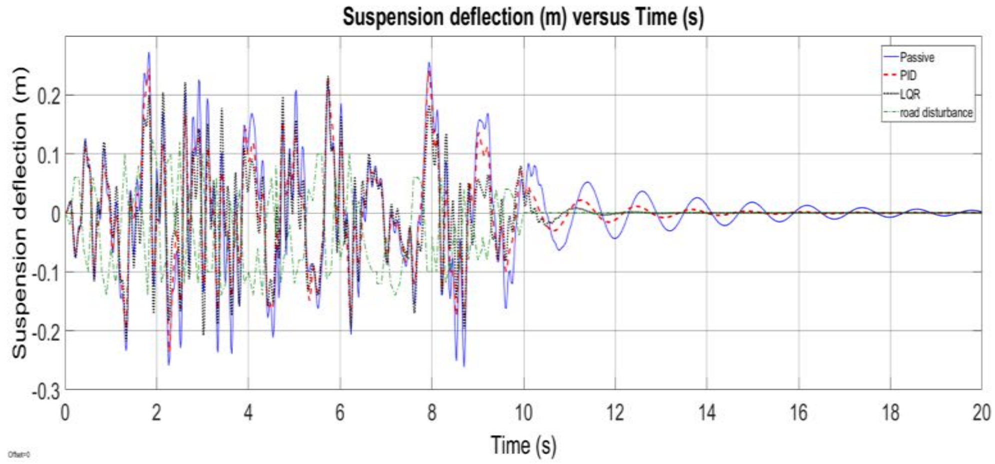
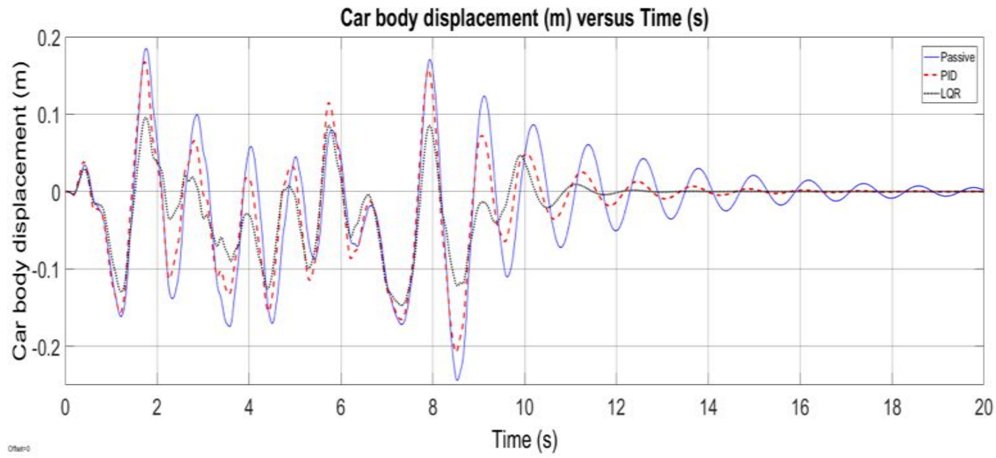



(c)

**Figure 6**: Vehicle performance of a car: (a) Suspension deflection; (b) Body displacement; (c) Wheel deflection with magnified view




(a)

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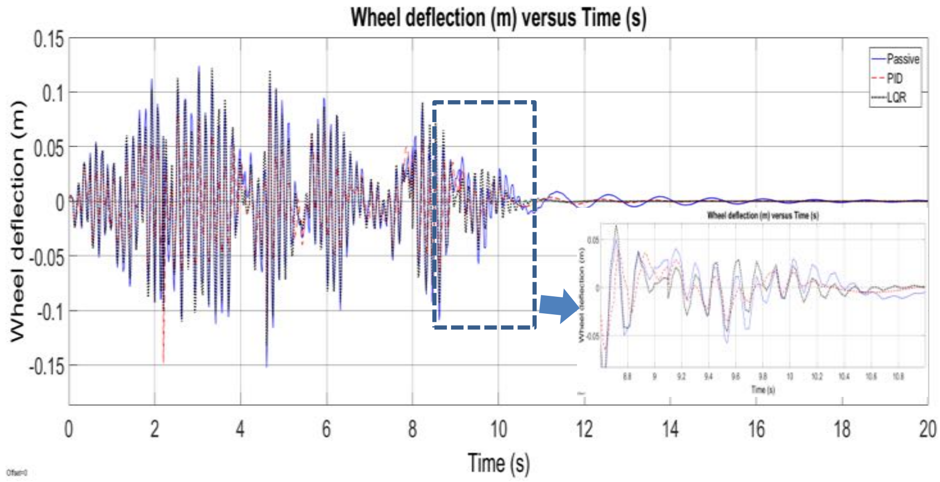


(c)

**Figure 7**: Vehicle performance of a bus: (a) Suspension deflection; (b) Body displacement; (c) Wheel deflection with magnified view

For further analysis, a detailed comparison between the performances of passive, PID and LQR are presented. Table 3 to Table 5 show the analysis of settling time and percentage of overshoot for bump scenario in term of suspension deflection, car body displacement, and wheel deflection. Meanwhile Table 6 to Table 8 demonstrate similar analysis but for road pavement scenario. The highest peak for each control method is taken for calculating the percentage of overshoot from the surface road.

**Table 3**: Analysis for bump scenario in term of suspension deflection

| **Vehicle** | Car | | Bus | |
|---|---|---|---|---|
| **Description** | Settling Time, $T_s$ | Overshoot, %OS | Settling Time, $T_s$ | Overshoot, %OS |
| Passive | 3.09s | 5.06% | 11.99s | 12.71% |
| PID | 2s | 3.33% | 8.57s | 10.60% |
| LQR | 1.01s | 6.01% | 3.91s | 10.23% |

**Table 4**: Analysis for bump scenario in term of car body displacement

| **Vehicle** | Car | | Bus | |
|---|---|---|---|---|
| **Description** | Settling Time, $T_s$ | Overshoot, %OS | Settling Time, $T_s$ | Overshoot, %OS |
| Passive | 3.55s | 18.44% | 12s | 21.85% |
| PID | 2.20s | 15.82% | 9.77s | 21.73% |
| LQR | 1.36s | 14.71% | 4.28s | 19.34% |

**Table 5**: Analysis for bump scenario in term of wheel deflection

| **Vehicle** | Car | | Bus | |
|---|---|---|---|---|
| **Description** | Settling Time, $T_s$ | Overshoot, %OS | Settling Time, $T_s$ | Overshoot, %OS |
| Passive | 4s | 1.10% | 12s | 2.57% |
| PID | 2s | 2.01% | 9.26s | 2.43% |
| LQR | 1.7s | 1.12% | 4s | 1.94% |

**Table 6**: Analysis for road pavement scenario in term of suspension deflection

| **Vehicle** | Car | | Bus | |
|---|---|---|---|---|
| **Description** | Settling Time, $T_s$ | Overshoot, %OS | Settling Time, $T_s$ | Overshoot, %OS |
| Passive | 6.29s | 26.17% | 18.18s | 27.28% |
| PID | 5.56s | 14.53% | 14.54s | 24.41% |
| LQR | 5.08s | 20.31% | 7.39s | 20% |

**Table 7**: Analysis for road pavement scenario in term of car body displacement

| **Vehicle** | Car | | Bus | |
|---|---|---|---|---|
| **Description** | Settling Time, $T_s$ | Overshoot, %OS | Settling Time, $T_s$ | Overshoot, %OS |
| Passive | 6.37s | 20.01% | 18.23s | 18.49% |
| PID | 5.67s | 13.65% | 15.67s | 16.77% |
| LQR | 5.57s | 11.49% | 11.57s | 9.56% |

**Table 8**: Analysis for road pavement scenario in term of wheel deflection

| **Vehicle** | Car | | Bus | |
|---|---|---|---|---|
| **Description** | Settling Time, $T_s$ | Overshoot, %OS | Settling Time, $T_s$ | Overshoot, %OS |
| Passive | 7.35s | 10.02% | 16.97 | 12.41% |
| PID | 7.27s | 7.44% | 9.23 | 8.74% |
| LQR | 5.54s | 8.69% | 8.65 | 8.19% |

By comparing the vehicle performances for both bus and car vehicles, it clearly shows that different types of vehicle designs and operating parameters exhibit different responses. This statement can be seen based on the simulation, where the oscillation for bus in passive mode takes longer time compared to car due to vehicle dynamic behaviour. This is because a bus uses the different type of suspension system due to its high load carrying capacities. Different height measurements of the two consecutive bumps indicate the output performances that will give different amplitudes. Besides, by differentiating the vehicle output response with and without controller, it can be seen that LQR control gives the best response in reducing the vibration and achieving fast settling time for all three parameters measured, which are the suspension deflection, car body displacement and wheel deflection for both car and bus vehicles.

## 5.0 CONCLUSIONS

This paper presents a LQR control that has been applied to an active suspension system of a quarter car model. The LQR control has been compared to the PID control and the existing passive suspension system where the LQR control shows the greatest performances. The vibration of suspension deflection, wheel deflection and car body displacement for all given scenarios have been monitored. The results show that the proposed controller improves the ride comfort by having less percentage of overshoot. Besides, in speed bump scenario, LQR provides the fastest settling time in less than 2 seconds for car and less than 5 seconds for bus. Meanwhile, for road pavement scenario, the settling times required are less than 6 and 12 seconds for car and bus vehicles, respectively.

## ACKNOWLEDGEMENTS

This work is financially supported by research grant GUP Tier 2 Universiti Teknologi Malaysia under Vot number 14J35.

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